And, under the same hypothesis—

$$\frac{\text{Probability of } b+ax_o \text{ exactly}}{\text{Probability of } b \text{ exactly}} = \frac{\{\psi_{ax_o-x_o}\}_{b+x_o}(\mathrm{B})_{b+x_o}}{\mathrm{B}}.$$

Now, if we had numerical values given for a and n, it might be possible for a limited number of values of x (say 1, 2, 3, 4, 5), to work out the above formula sufficiently for determining the above ratio approximately; and, in most cases, very few values worked out would be sufficient for any practical purpose, at least, if a is not less than 5 or 6.

On the Purchase of Life Assurance Policies as an Investment. By ARCHIBALD DAY, *Actuary of the London and Provincial Law Assurance Society, and Fellow of the Institute of Actuaries.*

[Read before the Institute of Actuaries, 2nd January, 1860.]

AN allusion made by Mr. Jellicoe to this subject, in a paper read before the Institute of Actuaries on the 28th November, has induced me to place on record the conclusions at which I had arrived after a somewhat recent consideration of the question.

It appears to me that the buyer of a policy by way of investment, in considering the price at which he should purchase, should come to his decision entirely independent of what might be the Office value. The two values have no connexion whatever one with another. In the one case the purchaser is about to make an investment; in the other, the Office proposes to redeem a liability.

The assumption is that the life on which the policy depends is an average one; or, at any rate, that the proposed purchaser knows nothing to the contrary. Speculations may, indeed, be made in the purchase of policies where the health of the life insured is known to be impaired; or prices exceeding the Office values may be paid for policies immediately prior to a division of profits, for the sake of surrendering them to the Office, after the declaration of bonus, at an increased price; but with these cases I do not now propose to deal. My present object is to show what sum should be given by a person contemplating a permanent investment and expecting to obtain a reasonable rate of interest for his money.

Viewing the question in this light, the purchaser of a policy has two things to consider—

1. The purchase of an absolute reversion payable on the death of a given life;

2. The grant of an annuity of the premium during the same life;

and, in fixing the price to be paid, he should be able to guard himself against loss in each transaction in the event of the life surviving beyond the average term.

To effect this, the reversion to the sum assured should be valued according to the modern formula, $1-d(1+\mathrm{A})$, d being at the rate of 5 per cent. (=·04762), and A representing the present value of an annuity on the life at a practicable rate of interest, say Carlisle $3\frac{1}{2}$ per cent. The value of the annuity of the premium should be taken at the same rate, being the price which would be demanded by a Society granting annuities to undertake the liability; the purchaser would then be safe in any event.

Value of sum assured $=\mathrm{S}\{1-d(1+\mathrm{A})\}=\mathrm{S}-\mathrm{S}d(1+\mathrm{A})$
Value of future premiums . . . $=\mathrm{P}(1+\mathrm{A})$
Value of policy . . . $=\mathrm{S}-(\mathrm{S}d+\mathrm{P})(1+\mathrm{A})$

EXAMPLE.—*Policy for* £500, *effected at Age* 40, *without Profits. Annual Premium,* £14. 7*s.* 6*d.*

At the expiration of	Value will be	One-third of Premiums paid.
10 years . .	Negative . . .	£47·916
15 „ . .	£10·00 . . .	71·875
20 „ . .	77·56 . . .	96·500
25 „ . .	133·31 . . .	119·792
30 „ . .	206·92 . . .	143·750
35 „ . .	256·67 . . .	167·710
40 „ . .	298·69 . . .	191·666

The popular idea of the value of a policy "without profits" appears to be one-third of the premiums paid. In the example given above, the value does not reach one-third of the premiums until more than 20 years have elapsed; bnt, after that time, the excess of the value over one-third of the premiums increases rapidly.

It may appear paradoxical that a policy should have no value for nearly 15 years, and then but an insignificant one; but although it may have a value to the holder and to the Office, it would be a bad bargain to a person buying it as a permanent investment.

The purchase *might* prove a good speculation if the purchaser paid even a higher price, but he would have no *certainty* of making 5 per cent. interest on his outlay.

Objection is taken to this mode of valuing the reversion, that, in practice, a person buying a reversion seldom or never does go to a Company to purchase an annuity of the interest. But if we sub-

stitute the old tabular value, $\mathfrak{A}$, a higher rate of interest must be assumed, and 6 per cent. will not then be deemed an exorbitant rate.

Valuing, then, the same policy by the formula $\mathfrak{A}-P(1+A)$, $\mathfrak{A}$ being at the rate of 6 per cent., and A, as under the former supposition, Carlisle $3\frac{1}{2}$ per cent.—

On the expiration of	Value will be
10 years	Negative
15 „	£17·687
20 „	77·65
25 „	127·59
30 „	188·23
35 „	245·38
40 „	286·725

results which do not differ materially from the previous values.

The foregoing has reference only to policies "without profits." When, however, they are participating policies, any reversionary bonuses that have been declared would of course be treated as additional sums assured, but allowance must be made for those that hereafter may be declared. This can be done only by the roughest approximation. If, by the constitution of the Office, a policy which becomes a claim in the interval between two divisions of surplus, is entitled to bonus in respect of the period from the last division, the value of the respective annual additions to the sum assured would be represented by $\frac{R_m}{D_m} \times b$; and though the rate of bonus per annum (b) on the sum assured is in every respect an *unknown* quantity, it may not, perhaps, be unfair to assume it at the same rate as the average of the previous additions (if any) to the policy, especially as great differences exist between the bonuses of different Offices, so that a uniform rate can hardly be assumed.

If bonuses do not accrue except at the end of each quinquennial or septennial period, for $\frac{R_m}{D_m}$ must be substituted a series of deferred assurances, $\frac{M_{m+n}+M_{m+2n}+M_{m+3n}, \&c.}{D_m}$, multiplied by the average addition obtained at previous bonuses. It is important, however, to know whether the mode of dividing the surplus allots, at each division, a bonus in respect of every year from the date of the policy, or whether merely in respect of the years completed since the previous distribution.

In conclusion, I give an example of three policies, the sale of

which I happened recently to witness at the auction mart. In the estimated values, allowance has been made for future bonuses, in accordance with the method above proposed.

Date of Policy.	Name of Office.	Present Age.	Sums assured and Bonuses already declared.	Premium.	Estimated Value.	Sold for
Sept., 1822	A	83	2,920	£100 0 0	£1,893	£1,960
„ „	B	83	2,812	92 8 7	1,828	2,000
July, 1834	C	68	2,326	82 13 4	734	960

On the Medical Estimate of Life for Life Assurance. By Stephen H. Ward, M.D.*

(Continued from page 263.)

HAVING thus glanced at different occupations as elements in the medical estimate of life, it may not be out of place to introduce the following table by M. Lombard, exhibiting trades in relation to consumption. In 1,000 deaths in each of the different occupations noticed, the following proportions were furnished by this disease:—

With vegetable and mineral emanations	176
With various dusts	145
With sedentary life	140
With workshop life	138
With hot and dry air	127
With stooping posture	122
With sudden movements of arms	116
With muscular exercise and active life	89
With exercise of the voice	75
Living in the open air	73
With animal emanations	60
With watery vapour	53

It may be further observed, that the better the condition of life, the less the liability to consumption. Marc d'Espine has proved that tuberculosis occasions 68 deaths per 1,000 among the rich, and 233 per 1,000 among the poor.

Residence.—There are but few points in regard to residence in this country which materially affect the value of life. Town life, though far healthier than it was a few years back, still falls far

* *See* Letter from Thomas Fraser, Esq., page 357.